Interactive Quantum Classifier Inspired by Quantum Open System Theory

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Abstract—Quantum theory has attracted people’s attention since it was proposed. Due to its unique advantages in information storage and processing, quantum information processing has become the most popular research field. Quantum theory also provides us with new methods or concepts for information manipulation and processing. The basic problems of classical physics are basically trying to be solved in a situation of isolation from the surrounding environment to reduce the complexity of the analysis problem, but the quantum system inevitably produces decoherence and establishes a close relationship with the environment such as entanglement, so the formal framework of quantum mechanics is inherently capable of depicting complex relationships. Based on the principle of quantum open system, a classifier under the formal framework of quantum mechanics is established to simulate the evolution process of open systems, that is, the interaction process between the target system and the environment. Specifically, we regard the features (or attributes) of the sample as environmental factors that affect the decision-making of the target system, and the target system can obtain the categories (or labels) of the sample through measurement. Based on this, we use the formal framework of quantum mechanics to establish a more natural and tighter correspondence between attributes and labels. Limited by the limitations of simulating quantum operations on classical computers, we conducted experiments on two lightweight machine learning datasets and compared them with mainstream classification algorithms. Experimental results show that the classification algorithm is better than the comparison models, and it also reveals the potential of the algorithm.

Index Terms—Quantum Machine Learning, Quantum Open System, Quantum Classification Algorithm

I. INTRODUCTION

Quantum theory is one of the greatest scientific achievements of human beings in the 20th century. It reveals the structure and properties of microscopic particles and makes people’s understanding of matter deep into the microscopic field. In recent years, quantum theory has also been widely applied in the information field, forming many interdisciplinary subjects [1], such as quantum information processing [2], quantum communication [3], quantum computing [4], etc.

The application of quantum mechanics in information science begins in the 1990s. In 1994, Shor [5] proposes a quantum algorithm (called Shor algorithm) that achieves $O((\log N)^3)$ time in the prime factorization of integers. This cause a sensation in the past, which theoretically reduces the time complexity of the prime factor decomposition to the polynomial time. Polynomial time is significant here, which means that the RSA encryption is no longer theoretically safe. In 1996, Grover [6] proposes a quantum search algorithm (called Grover algorithm) that reduces the complexity of the algorithm from $O(N)$ to $O(\sqrt{N})$. The Grover algorithm plays a role in the secondary acceleration of the classical algorithm, which significantly improves the efficiency of the search. Quantum information processing has a high degree of parallelism and leads to an exponential growth in storage capacity and acceleration, which significantly exceeds the conventional classical algorithms in terms of computational complexity and convergence speed [7]–[9]. Therefore, it has great advantages and strong vitality and has attractive research and application prospects.

Quantum theory also provides us with new methods or concepts for information manipulation and processing, such as quantum open system theory [10] or quantum measurement mechanism [11], etc. The basic problems of classical physics are basically trying to be solved in a situation of isolation from the surrounding environment to reduce the complexity of the analysis problem, but the quantum system inevitably produces decoherence and establishes a close relationship with the environment such as entanglement, so the formal framework of quantum mechanics is inherently capable of depicting complex relationships.

Based on the basic principles of quantum open system, we propose an Interactive Quantum Classifier (IQC). We consider the classifier as a physical system (target system) and consider the features (or attributes) of the sample as the environmental factors that affect the decision-making of the target system, and the target system can obtain the categories
Quantum machine learning has been rapidly proposed and received attention from academia and industry. Schuld et al. [22] introduce a quantum perceptron model imitating the step-activation function of a classical perceptron based on the quantum phase estimation algorithm. Chen et al. [23] propose a quantum neuro-fuzzy classifier for classification applications. It is a five-layer structure, which combines the compensatory-based fuzzy reasoning method with the traditional Takagi Sugen Kang fuzzy model. Nasios and Bors [24] introduce a new nonparametric estimation approach inspired by quantum mechanics. It considers the known location of each data sample and models their corresponding probability density function using the analogy with the quantum potential function. In addition, Refs. [25]–[31] also propose different methods and attempt to combine quantum computing with classical learning algorithms.

In the research of pure quantum mechanical learning algorithms, good results have also been achieved. Daskin [32] proposes a simple neural network constructed using quantum circuits and experimentally demonstrates the advantage of exponential acceleration compared to classical neural networks.
of the same structure. Cleve and Wang [33] consider the natural generalization of the Schrödinger equation to Markovian open system dynamics. They give a quantum algorithm for simulating the evolution of an \( N \)-qubit system. Liu et al. [34] propose a novel text classifier based on quantum computation theory. It considers the classification task as an evolutionary process of a physical system and builds the classifier by using the basic quantum mechanics equation. In addition, Refs. [15], [18], [35]–[37] also propose some meaningful work.

III. PRELIMINARY

A. Quantum open system

In order to illustrate the concept of quantum open system, here we need to distinguish between the different views of closed and open system in dealing with problems. The basic problems of classical physics are always sought to be resolved in an independent situation, i.e. isolated from the surrounding environment [38]. The benefit of isolating the surrounding environment is to reduce the impact of environmental noise on the system (such as environmental disturbances to the system), but isolation does not change the actual “nature” of the system. Therefore, that is why most researchers would consider this method of choosing isolate environment is harmless and helpful to solve problems.

Quantum systems make us re-evaluate the relationship between the target system and the environment. In the picture of quantum mechanics, the interaction between the target system and the environment will play a greater role than in classical physics. They usually lead to entanglement between two interacting systems [39], which changes the nature of the target system and fundamentally changes the phenomena we observe at the observation level [38]. The fragility of energy levels of quantum systems is emphasized by Zeh’s seminal papers [40], [41], who argues that macroscopic quantum systems are in effect impossible to isolate. When a quantum device is an open system, the interaction between the quantum device and the environment, i.e. the transmission of information, has a huge impact on the quantum device. It can be concluded that the interaction between the target system and the environment is the most critical feature to distinguish quantal systems from classical systems [42].

B. Quantum bits

Bits are the basic concept of classical computing and classical information. Quantum computing and quantum information are based on a similar concept, the quantum bit (qubit) [43]. In the classical world, we use 0 and 1 to represent the two states of the classical bits, respectively, but in the quantum world, we use state \( |0 \rangle \) and \( |1 \rangle \) to represent the two states of the qubits, respectively, where the notation “\(|·\rangle\)” is called the Dirac token. The difference between bits and qubits is that the state of the qubits can be between \( |0 \rangle \) and \( |1 \rangle \), i.e. a linear combination of the two states, often referred to as a superposition state, e.g. 

\[
|ψ⟩ = α|0⟩ + β|1⟩, \tag{2}
\]

where \( α \) and \( β \) are complex numbers. In other words, the state of qubits is a vector in a two-dimensional complex vector space, and the special \( |0⟩ \) and \( |1⟩ \) state are called their the ground state, which is a set of orthogonal basis that makes up this vector space.

We can measure to determine whether a quantum state is in the \( |0⟩ \) or in the \( |1⟩ \). But we can’t get the values of \( |0⟩ \) and \( |1⟩ \) at the same time by measurement. On the contrary, quantum mechanics tells us that we can only get limited information about quantum states. When measuring qubits, we get the probability that \( |0⟩ \) is \( |α|^2 \), then the probability of getting \( |1⟩ \) is \( |β|^2 \). Of course, \( |α|^2 + |β|^2 = 1 \), because the sum of the probabilities must be 1. We often use a superposition state of qubits, i.e. an equal probability amplitude superposition state,

\[
|ψ⟩ = \frac{1}{\sqrt{2}}|0⟩ + \frac{1}{\sqrt{2}}|1⟩. \tag{3}
\]

Qubit is actually a two-level system. In the same way, we can also construct multi-level systems and superposition states of its equal probability amplitude. Then the form of the N-level system is

\[
|ψ⟩ = α_0|0⟩ + α_1|1⟩ + \cdots + α_i|i⟩ \cdots \tag{4}
\]

where \( α_i \) are complex numbers and

\[
|α_0|^2 + |α_1|^2 + \cdots + |α_i|^2 = 1. \tag{5}
\]

And its superposition state of the equal probability amplitude is

\[
|ψ⟩ = \frac{1}{\sqrt{N}}|0⟩ + \frac{1}{\sqrt{N}}|1⟩ + \cdots + \frac{1}{\sqrt{N}}|i⟩ \cdots. \tag{6}
\]

C. Quantum gates

Classic computer circuits consist of wires and logic gates. Wires are used to transfer information between circuits, while logic gates are responsible for processing information and converting information from one form to another. In quantum information, there are quantum gates for processing quantum bits. The most common ones are the identity matrix \( (I) \) and the Hadamard gate \( (H) \). There are also

\[
X = σ_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \tag{7}
\]

\[
Y = σ_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}, \tag{8}
\]

\[
Z = σ_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \tag{9}
\]

which are called Pauli matrices.

D. System compounding and decomposition

Let \( V \) and \( W \) be the Hilbert spaces whose dimensions are \( m \) and \( n \), respectively, and then \( V \otimes W \) is a \( mn \)-dimensional space. The element of \( V \otimes W \) is a linear combination of the tensor product \( |v⟩|w⟩ \). In particular, if \( |i⟩ \) and \( |j⟩ \) are standard orthogonal basis of \( V \) and \( W \), then \( |i⟩|j⟩ \) is a base of \( V \otimes W \), and the commonly used abbreviations \( |v⟩|w⟩ \), \( |v, w⟩ \) or \( |vw⟩ \) are used to represent the tensor product \( |v⟩|w⟩ \). The same
method can also be applied to the compounding of operators. Let $A$ and $B$ be linear operators on $V$ and $W$, respectively. We can define a linear operator $A \otimes B$ of $V \otimes W$.

Perhaps the most profound application of reduced density matrices is as a tool for describing subsystems of composite systems. Reduced density matrices are very important, in fact, it is an indispensable tool for analyzing composite quantum systems. It was introduced by Landau in 1927 as the only density matrix that gives rise to the correct measurement statistics given the usual formalism that includes Born’s rule for calculating probabilities [43], [44].

Suppose there are physical system $A$ and $B$ whose compound state is described by the density operator $\rho^{AB}$ and the reduced density matrix for $B$ is defined as

$$\rho^A = Tr_B(\rho^{AB}),$$

(10)

where $Tr_B$ is an operator map, called partial trace with system $B$, and partial trace is defined as

$$Tr_B(\langle a_1 | a_2 \rangle \otimes | b_1 \rangle \langle b_2 |) = \langle a_1 | a_2 | Tr([b_1] \langle b_2 |),$$

(11)

where $\langle a_1 \rangle$ and $\langle a_2 \rangle$ are two vectors in state space $A$, and $| b_1 \rangle$ and $| b_2 \rangle$ are two vectors in state space $B$. The trace operation on the right side of Eq. (11) is a normal trace operation on system $B$, so

$$Tr([b_1] \langle b_2 |) = \langle b_2 | b_1 \rangle.$$ (12)

To complete the definition of partial trace, we need to add the requirement that the input is linear in Eq. (11).

IV. BASIC PRINCIPLE OF IQC

In this section, we will first give a general description of IQC, and then give a detailed description of the model architecture, data representation and parameter learning of IQC in the specific case of the two-category classification task.

As mentioned in the introduction, the main principle of IQC is to regard the classifier as a physical system and regard the whole classification process as an evolution process under the whole unitary operator. Since we only need to consider the relationship between the target system $Q$ and the environment $E$, we ignore their respective Hamiltonian. The Hamiltonian of the interaction between $Q$ and $E$ is

$$H^{int} = -\hbar g \sigma^Q \otimes \sigma^E,$$

(13)

where $g$ is the coupling constant, $g > 0$, and its magnitude reflects the strength of the interaction. The whole unitary operator of the composite system is

$$U(t) = e^{-iH^{int}t/\hbar}.$$ (14)

Combine the independent variables and use them as parameters of $\sigma^E$. The changed form is

$$U(\tau) = e^{i\sigma^Q \otimes \sigma^E(\tau)},$$

(15)

where $\tau = gt$. Since $Q$ is represented as a two-level system, i.e. qubit, $\sigma^Q$ can be represented by Eq. (7), (8) or (9). And since both the probability amplitude and phase of $Q$ need to be considered,

$$\sigma^Q = \sigma_x + \sigma_y + \sigma_z.$$ (16)

$\sigma^E(\tau)$ is an operator acting on $E$, and its extremely simple form can be defined as a diagonal matrix

$$\sigma^E(\tau) = \begin{bmatrix} w_1 \tau_1 & \cdots & \cdots & w_n \tau_n \end{bmatrix},$$

(17)

where $w_i$ is the parameter to be learned. Since the composition between quantum systems requires the tensor product, it will increase the computing resources exponentially. Here we use multi-level systems to represent the environment.

Since the target system has no bias towards the classification result in the initial situation, the initial value of the cognitive state should be a superposition state of equal probability amplitudes, i.e. Eq. (3), and its density matrix is

$$\rho_{cog} = |\psi Q\rangle \langle \psi Q|.$$ (18)

For the same reason, the environment is also initially a superposition state of equal probability amplitudes, Eq. (6), whose density matrix is

$$\rho_{env} = |\psi E\rangle \langle \psi E|.$$ (19)

Finally, we obtain the density matrix form $\varepsilon(\rho_{cog})$ of the target system after the action through

$$\varepsilon(\rho_{cog}) = tr_{env}[U(\rho_{cog} \otimes \rho_{env})U^\dagger]$$

(20)

and measure it to determine the probability value on each ground state, i.e. $|0\rangle$ is $p^0_0$ and $|1\rangle$ is $p^0_1$. According to the specific task, choose the desired result. At this point, the architecture of IQC is constructed.

A. IQC for two-category classification tasks

For the two-category dataset $\Gamma = \{ (\vec{x}_i, y_i) \}_{i \in N}$, the input data of the model is the vector $\vec{x}_i$, which together with the weight vector $\vec{w}$ constitutes the diagonal matrix

$$\sigma^E(\vec{x}_i) = \begin{bmatrix} w_1 x_{i1} & \cdots & \cdots & w_n x_{in} \end{bmatrix}.$$ (21)

Therefore, the whole unitary operator acting on the composite system is

$$U(\vec{x}_i) = e^{i\sigma^Q \otimes \sigma^E(\vec{x}_i)}.$$ (22)

The initial form of the composite system of the target system and the environment is

$$|\psi\rangle = |\psi_{cog}\rangle \otimes |\psi_{env}\rangle$$

(23)

where

$$|\psi_{cog}\rangle = \frac{1}{\sqrt{2}} |0\rangle + \frac{1}{\sqrt{2}} |1\rangle.$$ (24)
and
\[|\psi_{env}\rangle = \frac{1}{\sqrt{N}}|0\rangle + \frac{1}{\sqrt{N}}|1\rangle \cdots + \frac{1}{\sqrt{N}}|N-1\rangle. \quad (25)\]

Its density matrix is
\[\rho_{cog} \otimes \rho_{env} \quad (26)\]

where
\[\rho_{cog} = |\psi_{cog}\rangle \langle \psi_{cog}| \quad (27)\]

and
\[\rho_{env} = |\psi_{env}\rangle \langle \psi_{env}|. \quad (28)\]

From Eq. (20), \(p_0^2 \) and \(p_1^2 \) can be obtained, then
\[z_i = \begin{cases} 
0, & p_0^2 \geq p_1^2 \\
1, & p_0^2 < p_1^2
\end{cases}. \quad (29)\]

\(z_i \) is the classification result of IQC for \(\vec{x}_i \).

B. parameter learning of IQC

The loss function of the model is set to
\[loss = \frac{1}{2}(1 - p_1^2)^2. \quad (30)\]

As the model iterates over the training set, it makes \(loss \) smaller and smaller, and \(p_1^2 \) becomes larger and larger, and finally the classification result is determined by the higher probability value. Since the dataset has both positive and negative examples, it is necessary to add a coefficient \(c \) to distinguish between positive and negative examples, which is set to
\[c = z_i - y_i, \quad (31)\]

where \(z_i \) is the output result and \(y_i \) is the true result. When it is a positive example, \(y_i = 1, \ z_i \in \{0, 1\} \), then \(c \in \{1, 0\} \); when it is a negative example, \(c \in \{-1, 0\} \). Thus, the purpose of the unified loss function can be completed.

The update rule for the weights is
\[w_i = w_i - \eta \frac{\partial loss}{\partial w_i}, \quad (32)\]

where \(\eta \) represents the learning rate,
\[\frac{\partial loss}{\partial w_i} \approx (1 - p_1^2)x_i. \quad (33)\]

The final form of the weight update rule is
\[w_i = w_i - \eta(z_i - y_i)(1 - p_1^2)x_i. \quad (34)\]
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<th>Accuracy</th>
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The experimental performance is shown in Tab. II. The variance of the accuracy of the ten results under the two datasets, Iris and Wine, is shown in Fig. 2. It can be seen from the experimental results that IQC performs better in the three-category classification task and is superior to most comparison models. It shows that IQC has certain potential in solving multi-category classification tasks.

VI. Conclusion and Future Work

In this paper, we present an Interactive Quantum Classifier (IQC) inspired by quantum open system theory, and verify its feasibility and effects under the classic machine learning three-category datasets and compare it with some classic classifiers. We find that quantum phase is feasible to establish an interactive classifier based on the principle of quantum open system, and it has strong scalability, that is, it is very easy to integrate other influencing factors. Moreover, IQC performs well in classification on small sample training sets, which is superior to most classical classifiers. The time complexity of IQC depends on the optimization algorithm and the number of attributes. Moreover, simulating quantum computing on a classic computer always requires more computing resources and the computational efficiency is not too high, but we believe that quantum computers will solve this problem in the future. In fact, companies such as Google and IBM have made some exciting progress in this area.

The principle of quantum open system is easy to apply to the organization of complex systems and the evolution mechanism of quantum open system makes it easy to establish interactions between systems [40], [41]. IQC provides clues and ideas for constructing complex systems using the principles of open quantum system, and provides the possibility to integrate multiple factors for decision making, that is, the decision system is set to the target system, and the possible influencing factors are set as the environment.

In future work, we will study the optimization algorithm suitable for IQC to improve the training speed. Moreover, we will explore the introduction of quantum characteristics into IQC, so that the algorithm can play its unique quantum advantages.

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